

One-day Workshop on Kinetic Theory  
September 7, 2016 (Wednesday)

• **Title and Abstract:**

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**Regularity of stationary solutions to the linearized Boltzmann equations**

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**Abstract:** We consider the regularity of solutions to the stationary linearized Boltzmann equations in bounded  $C^1$  convex domains in  $\mathbb{R}^3$  for gases with cutoff hard potential and cutoff Maxwellian gases. Suppose that a solution has a bounded weighted  $L^2$  norm in space and velocity with the weight of collision frequency, which is a typical functional space for existence results for boundary value problems. We prove that this solution is Hölder continuous with order  $\frac{1}{2}^-$  away from the boundary provided the incoming data have the same regularity and uniformly bounded by a fixed function in velocity with finite weighted  $L^2$  norm with the weight of collision frequency. A smoothing effect due to the combination of collision and transport is used in the proof.

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**The initial boundary value problem for the Boltzmann equation with soft potential**

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**Abstract:** Boundary effects are central to the dynamics of the dilute particles governed by Boltzmann equation. In this talk, we will report some recent studies on both the diffuse reflection and the specular reflection boundary value problems for Boltzmann equation with soft potential. For the diffuse reflection boundary condition, we capture some new properties of the probability integrals along the stochastic cycles and improve the  $L^2 - L^\infty$  theory to give a more direct approach to obtain the global existence and time decay rate. As to the specular reflection condition, our key contribution is to develop a new time-velocity weighted  $L^\infty$  theory so that we could deal with the greater difficulties stemmed from the complicated velocity relations among the specular cycles and the zero lower bound of the collision frequency. These methods in the latter case can be applied to Boltzmann equation with soft potential for all other types of boundary condition.

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**Probability measures with finite moments and the homogeneous Boltzmann equation**

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**Abstract:** We characterize the class of probability measures possessing finite moments of an arbitrary positive order in terms of the symmetric difference operators of their Fourier transforms. As

an application, we prove the continuity of probability densities associated with measure-valued solutions to the Cauchy problem for the homogeneous Boltzmann equation with Maxwellian molecules. This is a joint work with Prof. Yong-Kum Cho, Prof. Yosinori Morimoto, and Prof. Tong Yang.

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## Global well-posedness and regularity of the Boltzmann equation with large amplitude initial data

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**Abstract:** The global well-posedness of the Boltzmann equation with initial data of large amplitude has remained a long-standing open problem. In this paper, by developing a new  $L_x^\infty L_v^1 \cap L_{x,v}^\infty$  approach, we prove the global existence and uniqueness of mild solutions to the Boltzmann equation in the whole space or torus for a class of initial data with bounded velocity-weighted  $L^\infty$  norm under some smallness condition on  $L_x^1 L_v^\infty$  norm as well as defect mass, energy and entropy so that the initial data allow large amplitude oscillations. Both the hard and soft potentials with angular cut-off are considered, and the large time behavior of solutions in  $L_{x,v}^\infty$  norm with explicit rates of convergence is also studied.

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## Nonlinear stability of the 1D Boltzmann equation in a periodic box

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**Abstract:** We study the nonlinear stability of the Boltzmann equation in the 1D periodic box with size  $1/\epsilon$ , where  $0 < \epsilon \ll 1$  is the Knudsen number. The convergence rate is  $(1+t)^{-1/2} \ln(1+t)$  for small time region and exponential for large time region. Moreover, the exponential rate depends on the size of the domain (Knudsen number). This problem is highly nonlinear and hence we need more careful analysis to control the nonlinear term.

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## Spectrum analysis of some kinetic equations

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**Abstract:** In this talk we analyze the spectrum structure of some kinetic equations qualitatively by using semigroup theory and linear operator perturbation theory. The models we consider include the classical Boltzmann equation for hard potentials with or without angular cutoff and the Landau equation with appropriate potentials.